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Strategic wage setting with complementarities

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Abstract

How does Firms' location impact the structure and the wage setting strategies? A simple theoretical model of firms' wage setting strategies helps answering this question. In this framework, firms have a symmetric production function, with two perfectly complementary labor inputs but face different access to a scarce type of worker. This paper shows that firms facing lower access to workers will pay higher rents and will hire less than firms with a better access to the scarce resource. In this model, the equilibrium is interior and unique and exists for a wide range of distributions of workers. I observe that effects of exogenous shocks on productivity, scarcity of the resource and differences in access to the latter are unambiguous and provide straightforward policy implications. This paper examines the connection between size and wages and completes the long lasting observation that bigger firms pay higher wages. Firms' location is a key determinant underlining the relationship between wages and firm's sizes, in particular, a big firm is predicted to pay lower wages to skilled workers compared to its smaller counterpart.

JEL Codes: L25, J31, J42, R12

Key words: Wage differentials, Urban economics, Labor economics

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1 Introduction

How does firms' location drive wage setting strategies and wage inequalities between and within firms? This question is related to issues for rural firms to hire managers, in opposition with firms located in urban areas that have better access to skilled labor. Indeed, lack of reputation or remote workplaces can dissuade skilled workers to apply to these firms. Ability to recruit managers is thus not only an economic problem but has consequences on the appeal of territories that are shaped by the dynamism of the area. As the labor market shapes the structure of firms and the inequalities between areas, determining the relationship between geography, firms' structure and wages is of key relevance.

This thesis seeks to answer a number of associated questions. What are the consequences of these differences in access to pool of workers? How do differences in commuting costs and densities of workers drive wage differences, within and across firms?

The economic literature has been interested in the interaction between geography, wages and firm size since a long time, mostly but not exclusively through empirical studies. These previous works, summarized in a dedicated section, allow to answer some specific mechanisms pertaining to a global question.

Tackling these issues with a theoretical framework allows to determine the individual impact of each component in the geography-wages-firms system. In the model developed in this paper, firms compete for labor, the technology of production depicts perfect complementarity between skilled and unskilled labor, following a broader framework on rent-sharing. Workers are distributed along a line where the relative mass of skilled workers varies along this line. Skilled workers are scarce and some unskilled workers face unemployment. Imperfect and asymmetric competition drive the results of the paper and allow to define the interactions of geography, wages and firms' sizes. Asymmetry in competition arises from a difference in access to the pool of workers, a firm located in a area where the scarce resource sets the wage differently from a better located firm.

The model offers three main results that are of interest for theoretical or empirical purposes.

First equilibrium existence and uniqueness hold under very reasonable assumptions that can be observed empirically. This strength of the result lies in the surprising simplicity of the conditions required. My paper allows safely to assess that the rest of the results of the paper are consistent and observable empirically.

Second, as firms face a scarcity of workers, differences in access to pool of workers (through location) drive the structure of wages and firms. This paper determines that firms with a greater endowment of workers face reduced competition pressure and are allowed, simultaneously, to grow bigger and offer a lower remuneration to the scarce resource. This result consistent with the rent-sharing theories, nuances the observation that bigger firms pay higher wages by assuming homogeneity in skill composition of firms.

Finally, the comparative statics are helpful to determine sector or industry specific impacts on equilibrium characteristics. Among the range of parameters I notably include differences in productivity of firms, the degree of asymmetry in access to pool of workers, the change in demand for worker's skills. These results are consistent with the existing empirical observations and complete their analysis.

The paper is organized as follows In section II, I provide a literature review of different strands of literature that recoup the research question raised by this paper. I describe various strands of literature that the evolution of differences in wages, through different factors. In section III, I develop and solve the model. The main results of the paper are all contained in this section. Section IV consists of concluding remarks on the paper. Finally, the Appendix provides proofs for the main results of the paper with extensive interpretation of the results.

2 Firms' sizes, wages and geography

Big firms paying better The seminal work of Brown and Medoff (1989) shows a wage premium offered by bigger firms. In their article, the authors already emphasised the role of other factors, as the skill of workers or the working conditions for instance. Still one can determine the most relevant factors driving the differences in wages between workers and firms, closely related to the model developed below. Namely, worker's individual characteristics, geography, compensating differences or the degree of competition are often retained as relevant determinants of wage differences.

In this section I will detail the extant research of each strand of literature aforementioned.

Individual characteristics.

Difference in firms' sizes is accompanied by difference in composition or structure, as denoted by Mincer (1974) the individual characteristics of workers play the biggest role to explain the differences in wages between and within firms. The most straightforward examples being education and job experience driving wages upwards. However, position in the firm and its characteristics often fit with workers' individual characteristics even when the precise skill or education level is not available in data.. The absence of heterogeneity between workers of same skill still allow to determine the differences in rent through the scope of differences in demand for skilled or unskilled labor. In my model, scarcity in supply of a specific skill yields a premium in rents for workers having this skill. In line with the observation by Acemoglu and Autor (2011), my models encompasses skill-biased sharing of rents in a spatial framework, introducing firms and workers locations. The interaction of geography and scarcity of skilled labor determines a specific evolution of rent differences between workers of same and different skills.

Geography.

Spatial models of labor saw a resurgence lately with the work of Combes et al. (2008). In their model, after controlling for workers' individual characteristics, the authors find a positive relationship with wages and density alongside a skill-sorting in France. The framework proposed by the authors is the first one uniting the role of skills, local endowments and interactions between workers and firms. Also notable the works of Groot et al. (2014) study adopt the same approach applied on Netherlands micro-data.

In the model developed below, workers are exogenously distributed along a line, formulating this local endowment solely drives the distribution of workers. Finally the technology of production drives the demand for labor and competition and thus the wages offered by firms.

Secondly, spatial mismatch between job opportunities and households location can drive wages of low income workers down and even lead to unemployment. First defined by Kain (1968) , this hypothesis defines the relationship between unemployment of African American in center of urban areas and location of firms in the suburbs. The recent survey of Gobillon and Selod (2021) precises the theory of spatial mismatch and the author define mechanisms leading to spatial mismatch. In addition to that, the authors precise methods to lead empirical tests related to the spatial mismatch hypothesis.In absence of information on racial characteristics, the distance from potential workplaces, the cost of housing and job search issues can still lead to a disconnection between households and workplaces.

The approach used does not model some key causes of the disconnection between low skilled workers and workplaces. However the model still exerts consequences of spatial mismatch, namely, unemployment and poverty in disconnected neighborhoods. This result, independent of racial or gender identity of workers is driven by commute costs and distance to workplaces. At the same time, greater spatial mismatch is accompanied by lower wages for currently employed workers. This effect is greater for bigger firms and reduces the differences both in firms' sizes and wages offered.

Compensating differences.

The study of spatial models relate to the literature on compensating differences, surveyed by Smith (1979) and Rosen (1986), firms willing to hire workers need to compensate unpleasantness of the job.

In particular, firms offer higher wages to remunerate commuting costs of workers, in a an intra-urban model Moses (1962) is the first to relate wages and geography in a theoretical framework. Moses, in his paper demonstrates that firms compensate workers that live farther away from the firm. However the author mentions the absence of the impact of the distribution of the population and the equilibrium size of firms. This is precisely a core component of my model and I show that differences in distribution of workers drive inequalities in wages for all workers and in sizes between firms for a given production technology.

The role of location on quality of life has been investigated in the literature with the works of Roback (1982), Gabriel et al. (2003) or Duranton and Monastiriotis (2002). The core of the paper focuses on the role of commute costs and differences in distribution of workers that determine the wage gradient.

The work of Timothy and Wheaton (2001), using US micro-data, allows to determine the wage gradient associated with commuting costs and firm location. In this paper the authors determine a positive relationship between wages and the average commute time. The model in my paper includes the role of commuting costs on wage setting strategies. Namely the comparative statics of the commuting cost unveil interesting properties, as differences in rents offered to workers increase with commuting costs as observed empirically. Not only replicating empirical results, my model shows a reduction in individual wages after a fall in mobility of workers. For some workers, being less mobile is accompanied with a fall in wages as firms face less competitive pressure.

Degree of competition.

The degree of competition between firms when competing for workers or on the final good market greatly alters the wage setting strategies of firms. In line with the Industrial Organization literature, assuming commuting costs leads to differentiated preferences for firms. In this context, firms wage setting is also affected by competition. A firm located in a denser area will have less competitive pressure to raise wages as she can hire more workers, for the same wages the other firm pays to its workers. Empirical works on monopsony, as reviewed by Manning (2011) focuses on wage differences derived from different competitive frameworks. The main mechanism of these series of models is the rent-sharing between firms and workers explaining the wage differences. The idea that a more competitive market leads to a lower (higher) rent for firms (workers) . In the framework I develop, firms face a different competitive pressure due to asymmetries in access to the pool of workers. A firm facing a small pool of available workers will have more incentives to increase the rent it offers to workers. Furthermore the results on differences in productivity recoup empirical results that shocks transmit partially to wages through imperfect competition.

3 Competition for labor with input complementarity

3.1 Model Setup

There are two types of workers in the economy, L workers and H workers. These two types differ in skills or education, implying that they have different jobs in the firm.

Moreover, workers are characterized by a second dimension, their location on a Hotelling line. The utility of a i worker located in $x \in [0,1]$ from signing in firm j, $U(x|j$, is the following:

$$\forall\ i \in \{H;L\},\ \forall\ x \in [0;1],\ U_i(x|A) = w_A^i - tx$$
$$\forall\ i \in \{H;L\},\ \forall\ x \in [0;1],\ U_i(x|B) = w_B^i - t(1-x)$$

The participation constraint of this worker is the following:

$$\forall\ i \in \{H;L\},\ \forall\ x \in [0;1],\ \exists\ j,\ U_i(x|j) \geq U_i(x|-j) \geq 0$$

$M(x)$ represents the cumulative mass of H workers in $[0;x]$, $m(x)$ is the instantaneous density of H workers at x.

Since I focus on the relative scarcity of H workers, I normalize the density of L workers to 1 along the line.

The parameter t represents the transportation cost of workers, in particular it is the cost for a worker living at $x = 0$ to work in a firm located at $x = 1$. This commute is assumed linear in the model. In the standard Hotelling model, the transportation cost can represent a physical cost of moving from one place to another or the subjective cost of adapting to a specific product, the model developed in the paper does not focus on the final good market but on the competition for workers. It is possible to reformulate commute costs incurred by workers as costs of working in a firm of a given reputation. Workers are then differentiated for their preferences in firms' notoriety. This paper will mostly interpret the results as geographic distance and not to subjective preferences over firms' reputations. Indeed, introducing subjective preferences and assuming a particular distribution of the latter limits the relevance of the model as a general framework. The results of this paper are convenient to interpret in an urban model, where x represents a physical location.

Two firms are located at each extremity of the $[0; 1]$ line: firm A is located at 0 and firm B is located at 1. The variable x denotes workers' location on the line. The firms are not able to determine where each worker is individually located but has information on the mass of workers in any given interval. As a result each firm j , proposes a rent w_j^i to $i \in \{H, L\}$ to all workers of a given type.

The wage encompasses utility relevant dimensions of the job, namely the level of responsibility of the position, the work life balance, and other amenities offered by the firms' contracts. A higher level of utility might not be translated to a higher nominal wage. For instance, a worker can accept a lower wage if it is in his interest to have more freedom on the job or more diversified tasks. This versatile definition of the wage accommodates for the different frameworks that big and small firms offer. This particular definition of the wage allows for a better fit with real life considerations of workers and firms.

Let h and l be the respective mass of H and L workers, the production function is defined as:

$$R(h, l) = k \min\{h; \alpha l\} \tag{1}$$

This production scheme, common to both firms, depicts H and L workers as perfect complements.

This representation lends itself to a natural interpretation, for the services and industry sectors. Suppose that each L worker is required to be supervised by no more and no less

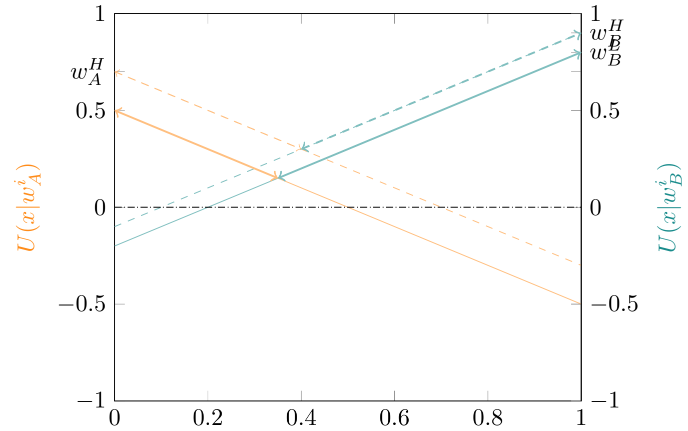

Figure 1: Employment decisions of agents for $(w_A^L, w_A^H, w_> B^L, w_B^H) = (0.5, 0.7, 0.8, 0.9)$. Colored dashed lines represent L workers, whilke full lines represent H workers. The black dotted line in the middle is the reservation utility of the workers. The colors are associated with firms, orange for A and teal for B. Finally the colored arrows represent the workers hired by each firm.

than $1/\alpha$ H workers, it is the 'supervision rate'.

In the service sector, it is relevant is to assume that H workers provide tasks or contracts that α L workers execute.

In the industry sector, L workers are overseen by H with a supervision rate of $1/\alpha$.

As such any deviation from an ideal 'supervision rate' will not lead to increase in production. In the case where there is an under-provision of contracts ($h < \alpha l$), some L workers are idle on the job. Conversely, if $h > \alpha l$ contracts accumulate and cannot be achieved. One could view this as extreme assumption for firms lack of flexibility. However the goal of this assumption is to represent differentiated demand for one type of labor, that depends on a single, tractable and observable parameter α.

The timing of the game is composed of two steps. First, firms post wages they offer publicly, then the workers apply and enrol in their preferred firm. The firm is able to distinguish between L and H workers when they apply.

The profits of the firms are:

$$\Pi_A = k \min(h_A; \alpha l_A) - w_A^L l_A - w_A^H h_A \tag{2a}$$

$$\Pi_B = k \min(h_B; \alpha l_B) - w_B^L l_B - w_B^H h_B \tag{2b}$$

Where $\forall\ i \in \{h; l\},\ \forall\ j \in \{A; B\},\ i_j$ is the number of workers i hired by firm j.

Given the Production function, we are required to distinguish 2 situations in order to determine the profits. As the results hold symmetrically with a relabelling, only one case will be studied.

Assuming that $M(1) < \alpha$, in this case some L workers will be unemployed, and all H workers will be employed in equilibrium. Consequently, the L workers job market is separated in 2 and firms hire locally. The rent of L workers is then fully determined by the size of the firm, and how many H workers it employs. The only incentive for firm j to set $w_j^L > 0$ is to compensate the unskilled workers for commute costs.

One can define $\underline{x} = \frac{w_A^H}{t}$, and $\overline{x} = \frac{t - w_B^L}{t}$. No L worker in $\left[\underline{x}; \overline{x}\right]$ is employed. Firms compete in rents only for the H workers, meaning that only H workers can receive positive utility offers from both firms.

Given the production function, each firm will hire L workers in fixed proportion to their number of H workers. Such that $\alpha\underline{x} = h_A$ and $\alpha\overline{x} = h_B$.

The total expenditure in L workers is equal to:

Lemma 1 (Firms' total expenditure for L workers)**.**

$$w_A^L l_A = \frac{t}{\alpha^2}(h_A)^2 \tag{3a}$$
$$w_B^L l_B = \frac{t}{\alpha^2}(h_B)^2 \tag{3b}$$

When hiring a H worker, a firm needs to increase the rent she offers to L workers by t/α while the other firm decreases the rent offered to L workers by the same amount.

Due to the distribution of L workers, the marginal cost of hiring a new L worker is convex.

As no discrimination between workers of each type is possible, they all receive the same wage in equilibrium. However, the closer a worker is from the working place, the less costly it is for him to commute. The marginal increase in wage offered to all L worker is equal to the marginal increase in commute cost of the last L worker hired. As such the total expenditure is convex.

Due to perfect complementarity of labor inputs, the firms only have one adjustment variable, namely wages offered to H workers.

Whether a firm is constrained locally by the supply of L drives the shape of the marginal cost of hiring L workers, relaxing this local constraint keeps most of the results of the model unchanged.

Given the initial conditions of scarcity of H workers, one can prove that no H worker is unemployed in equilibrium:

I define unemployment for some H workers as follows:

$$\exists\ \{\underline{x}^H;\overline{x}^H\},\ \underline{x}^H = \frac{w_A^H}{t},\ \overline{x}^H = \frac{t - w_B^H}{t},\ \underline{x}^H < \overline{x}^H \tag{4}$$

In $[\underline{x}^H;\overline{x}^H]$ there are no H workers employed. With this definition and using the first order conditions of 2, one can describe minimal conditions for full employment of H workers.

Lemma 2 (Full employment of H workers)**.** *Under conditions of productivity, some H workers will be unemployed in equilibrium, they are located in $[\underline{x}^H;\overline{x}^H]$:*

$$\forall\ M(\cdot)\ \textit{log-concave},\ \exists!\ k_{min}, \forall\ 0 < k < k_{min}\ \exists\ \{\underline{x}^H;\overline{x}^H\} \in [0;1]^2,\ \underline{x}^H < \overline{x}^H \tag{5}$$

For the rest of the paper we will restrict our scope to cases where all H workers are employed, $k > k_{min}$. Indeed, if this minimal condition of k is not full-filled, firms do not compete for any type of worker, restricting the scope of analysis.

Given that, it is now possible to determine the location of the H worker that is indifferent between working in firm A and working in firm B. This worker will be now on be called the indifferent worker.

The indifferent H worker located at $\tilde{x}^H$ is determined such that:

$$U_H(\tilde{x}|A) = U_H(\tilde{x}^H|B) \equiv \tilde{x}^H = \frac{1}{2} + \frac{w_A^H - w_B^H}{2t} \tag{6}$$

As such one can immediately see that wages are bounded by the following:

$\forall j\ \in [H;L],\ w_{-j}^H + t \geq w_j^H \geq w_{-j}^H - t$

Finally, the profits of the two firms are the following:

$$\Pi_A = kM(\tilde{x}^H) - w_A^H M(\tilde{x}^H) - \alpha^{-2}t\Big(M(\tilde{x}^H)\Big)^2 \tag{7a}$$

$$\Pi_B = k\Big(M(1) - M(\tilde{x}^H)\Big) - w_B^H\Big(M(1) - M(\tilde{x}^H)\Big) - \alpha^{-2}t\Big(M(1) - (M(\tilde{x}^H)\Big)^2 \tag{7b}$$

The first order conditions are the following:

$$k - w_A^H - 2\alpha^{-2}tM(\tilde{x}^H) = \frac{M(\tilde{x}^H)}{\frac{\partial M(\tilde{x}^H}{\partial w_A^H}} \tag{8a}$$

$$k - w_B^H - 2\alpha^{-2}t\Big(M(1) - M(\tilde{x}^H)\Big) = \frac{\Big(M(1) - M(\tilde{x}^H)\Big)}{-\frac{\partial M(\tilde{x}}{\partial w_B^H}} \tag{8b}$$

The right hand side of each expression is the inverse of the rate of increase of firm's size when it increases the rent of H workers, this can then be interpreted as the required increase in rents to increase current size of the firm by 1% at location $\tilde{x}^H$. It corresponds to the inverse of the hazard (reversed hazard rate) of $M(\cdot)$, and will provide associated properties when discussing the comparative statics with respect to $M(\cdot)$ in section 3.4.4.

3.2 Benchmark case

In this short section, I provide a simple equilibrium result, where firms have a symmetric location. This case stands as a benchmark, where firms are assumed to face the same supply of labor and motivates the development of the core of my paper.

Suppose that the distribution of workers is symmetric around $x = 0.5$,

$$\forall\ x \in [0;1], M(x) \text{ is symmetric around } x = 0.5 \text{ if and only if } M(x) = M(1) - M(1-x)$$

Given (8) and the symmetry property of $M(\cdot)$ I can prove that a symmetric equilibrium in pure strategies exists, $w_A^H = w_B^H = w^H$ Given that, it must be that $\tilde{x}^H = \frac{1}{2}$, using (8a) and (8b), they collapse to the following necessary and sufficient condition for a symmetric equilibrium to exist:

$$k - w^H - \frac{2t}{\alpha^2}M(\frac{1}{2}) = \frac{M(\frac{1}{2})}{\frac{\partial M(\frac{1}{2})}{\partial w^H}}$$

So a symmetric equilibrium in pure strategies exists under standard condition of full employment of H workers. Indeed, it is always possible to adjust w^H to attain the condition above. In this framework, this equilibrium might not be unique, it holds as a relevant starting point to deviate from by relaxing the symmetry assumption.

However, assuming this symmetry of location of firms is, I believe, non standard and accommodates for only few real life situations. Thus, defining a general framework with specific distribution of workers is closer to reality. As the skilled workers are not uniformly distributed in areas of employment, defining a case in opposition to this benchmark case holds as a starting point to answer the research question.

3.3 Asymmetric Access to scarce workers

In this section we will address the issue that firms do not enjoy a symmetric geographic position have a different access to the pool of H workers. Indeed, deviating from the benchmark case allows to define variation in rents within and between firms that arise from the competitive framework.

In that sense, I now suppose that $M(\cdot)$ is strictly convex. The density of H workers keeps increasing as we approach $x = 1$. This confers an advantage to firm B that now has a better access to H workers than firm A.

The following figure depicts the instantaneous mass of L and H workers as an example of this case For $m(x) = 0.8x$, this figure pictures an example of the advantage firm B benefits from. The total mass of H workers is $M(1) = 0.4$.

On both of (8) the left hand side is decreasing in the firm's j decision variable (w_j^H). To guarantee existence of an interior equilibrium, we need to find conditions for which the right hand side of each equation is increasing in firm's rent offered to H workers.

Proposition 1 (No Corner equilibrium)**.** *There is no corner equilibrium where any firm is the unique employer in the market.*

This first result means that firms do not adopt predatory strategies that push the concurrent out of the labor market. This result is the first step that ensures the uniqueness of the equilibrium.

Proposition 2 (Existence and Uniqueness)**.** *If $M(\cdot)$ is log-concave then the interior equilibrium exists and is unique.*

The proposition asserts that if $M(\tilde{x}^H)$ is log-concave then the equilibrium exists and is unique. Intuitively the log-concavity assumption can be interpreted as a decrease in the growth rate of the firm size when rents offered by the firm increase. Analytically, this condition corresponds to the right hand side of (8a) being increasing in w_A^H .

If, for any given size, the marginal rent increase needed to increase the size of the firm by 1% is increasing, then the equilibrium is unique. One can note that the symmetric condition is automatically satisfied for firm B when $M(\cdot)$ is convex, as the right hand side of (8b) is increasing in w_B^H when $M(\cdot)$ is convex.

Finally, if $M(\cdot)$ is log-concave, then the second order conditions holds.

Now that we have conditions for existence and uniqueness of the equilibrium, we can also determine properties on equilibrium rents.

Proposition 3 (Equilibrium Characterisation)**.** *For any convex distribution of H workers, then in equilibrium we have*

- *Firm A pays higher rents for H workers than firm B: $w_A^H > w_B^H$*
- *The wage for L workers is greater for who works in B rather than A: $w_A^L < w_B^L$*
- *The price paid by the firm for the scarce resource is higher: $\forall j,\ w_j^L < w_j^H$*
- *Firm A is smaller than B: $M(\tilde{x}^H) < M(1) - M(\tilde{x}^H)$*

This result nuances results on large firms wage premiums. Here, firms having a better location can grow bigger while paying lower wages. As the concentration for a type of workers increase, firm B suffers less from competitive pressure and can set wages offered at a lower level. This results contrast with the extant literature on the size-wage gap, Brown and Medoff (1989). Geographic constraints can have effects offsetting partially composition and structural effects on large firms wage premium. In this model, firms are allowed to grow in size if they have a good local endowment in the scarce resource.

The nature of the compensation is not limited to monetary compensation the firm offers. The variable w_j^i encompasses the wage and non-wage benefits the firm offers. Still, the

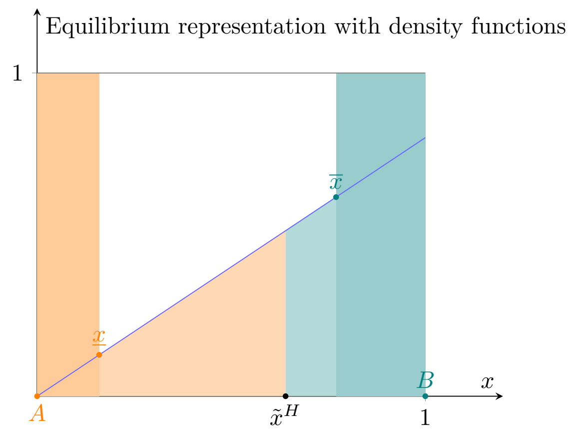

Figure 2: Equilibrium characterization for $m(x) = 0.8x$, the teal (orange) shaded area represents the mass of workers hired by firm B (A) located at $x = 1$, ($x = 0$). The darker areas represent the mass of L workers.

worker can receive advantages, for instance by the level of freedom while operating tasks. In that sense, defining w_j^i as a rent is a possible alternative. In this context, rents and wages will be used interchangeably for the remaining of the paper.

3.4 Comparative statics

In this section, I will describe the comparative statics of the equilibrium. The results are described for individual wages and the wage differences between and within firms.

First, the effect of an increase in productivity can arise through two parameters in the model, α and k. While k represents the value of the good or service provided, α represents the efficiency of the internal organization of the firm. The less H workers are required to operate, the less costly it is for her to produce a given output. The implication of a positive shock on productivity are different whether it applies to α or k. Due to the scarcity of H workers in the market, the firms and workers see their rent evolve differently after a shock on either of the parameters, changing the interpretation depending on the origin of the productivity shock.

Second, the effect of an increase in commute costs for each worker will be presented. The relevance of this results as they complete empirical observations of Timothy and Wheaton (2001) relative to the increase of wages when distance to workplace increase. Defining the tension between market power and mobility of workers allows to derive dynamics of individual's wage setting in addition to wage gaps already existing results.

Finally, the distribution of the H workers along the line allows to determine the effect of the extent of the asymmetry in access to workers. Indeed, for a given mass of skilled workers, ordering distributions allows to determine the extent of the wage differences between and within firms. A simple approach is used to implement this change in distributions using properties of stochastic ordering.

Using (6) and (8), it is possible to determine the comparative statics of the wage gaps between and within firms.

$$\Delta w^H = \big(M(1) - 2M(\frac{1}{2} + \frac{\Delta w^H}{2t})\big)\big(\frac{1}{M'(\frac{1}{2} + \frac{\Delta w^H}{2t})} + \frac{2t}{\alpha^2}\big), \tag{9}$$

where $\Delta w^H = w_A^H - w_B^H$.

Differentiating this expression allows to determine the comparative statics of the wage gaps between the firms.

3.4.1 Effect of an increase in productivity:

An increase in productivity $\mathrm{d}k > 0$ in both firms has no effect on the wage gap of H workers. In this imperfect competition framework, symmetric shocks on productivity fully transmit to the rents of H workers.

This result on wage gaps begs for a thorough analysis, especially if firms face different productivity shocks. Shocks affecting one firm only can affect the rent splitting in firms due to the imperfect competition and the local differences in pool of workers.

Denoting k_j the productivity of firm j , such that $k_A \neq k_B$, then one can rewrite first order conditions as follows

$$\Big(k_A - w_A^H - 2\alpha^{-2}tM(\tilde{x}^H)\Big) = \frac{M(\tilde{x}^H)}{\frac{\partial M(\tilde{x}^H)}{\partial w_A^H}} \tag{10a}$$

$$\Big(k_B - w_B^H - 2\alpha^{-2}t\Big(M(1) - M(\tilde{x}^H)\Big)\Big) = \frac{\Big(M(1) - M(\tilde{x}^H)\Big)}{-\frac{\partial M(\tilde{x}^H)}{\partial w_B^H}} \tag{10b}$$

An expression of the wage gaps in the spirit of (9) is

$$\Delta w^H = \left(M(1) - 2M(\frac{1}{2} + \frac{\Delta w^H}{2t})\right)\left(\frac{1}{M'(\frac{1}{2} + \frac{\Delta w^H}{2t})} + \frac{2t}{\alpha^2}\right) + \Delta k \tag{11}$$

Where $\Delta x = x_A - x_B$ This equation allows to determine the first result on comparative statics. The equations associated with each individual rent are provided in the appendix, the process, however is the same.

Proposition 4 (Increase in Productivity)**.** *An increase in productivity has different impacts whether the shock is global or specific to one firm:*

- *An increase in productivity for both firms have no impact on the wage gap of H workers in equilibrium. Still, within firms the wage gap increases in favor of H workers. Consequently H workers fully benefit from this increase in productivity: if* $\mathrm{d}\Delta k = 0,\ \mathrm{d}\Delta w^H = 0$ *and* $\forall\ j \in \{L; H\},\ \mathrm{d}k = \mathrm{d}w_j^H$
- *An increase in the productivity gap in favor of firm j leads to a greater increase in rents of H workers in firm j than H workers in the other firm:* $\mathrm{d}\Delta k > 0,\ \mathrm{d}\Delta w^H > 0$ *and* $\mathrm{d}\Delta k < 0,\ \mathrm{d}\Delta \mathrm{w}^{\mathrm{H}} < 0$

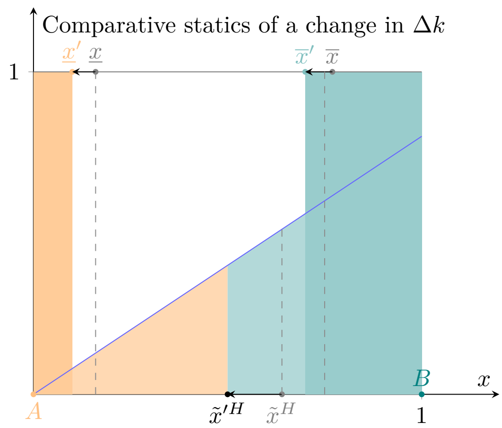

Figure 3: Effects of $\Delta k = 1$, with $M(1) = 0.4,\ \alpha = 2,\ t_L = 1,\ t_H = 2,\ k_A = 10,\ k_B = 11$
The figure above depicts the change in equilibrium after a positive shock on the productivity of firm B from 10 to 11. The dotted lines represent the former equilibrium.

To describe and analyze this result, I decompose the total effect into, first, a direct effect of the initial change in productivity. Then the shift in size of each first completes to determine the total effect by offsetting partially the first effect. The economic explanation for this result stands in two arguments: First, if a firm have a greater productivity than the other is willing to increase the rents she gives to H workers. Then, after the change in wage gap, the indirect effect of change in sizes partially offsets the first effect. It pushes the rent of H workers down (up) for the most (least) efficient firm as it grows (falls in size).

3.4.2 Increase in α

Across industries or countries, it is likely that the supervision rate varies. For instance, in some countries, a task can be automated thus requiring less low-skilled workers than other places in the world. determining the effect of a change in this parameter allows to compare sectors and industries. Technologies of production do not depict the same requirements for skilled labor, α then allows to derive prediction across industries.

A positive shock on α increase the tension on the H workers labor market. Already in scarce quantity, an increase in requirements of supervision decreases the demand for L workers for both firms, it is then less costly for bot h to hire L workers.

Proposition 5 (Increase in supervision rate). *An increase in the supervision rate* $\mathrm{d}\alpha > 0$ *has the following effects*

- *An increase in rents of H workers for both firms:* $\mathrm{d}w_A^H > 0,\ \mathrm{d}w_B^H > 0$
- *The increase is greater for H workers in firm B:* $\mathrm{d}w_B^H > \mathrm{d}w_A^H$
- *The wage gap of H workers between firms decreases, and the size of firm B increases:* $\mathrm{d}\Delta w^H < 0,\ \mathrm{d}M(\tilde{x}^H) < 0$

In the spirit of the analysis of the productivity gap, It is possible to decompose the total effect in two effects. First a direct effect on firms is associated with the fall in the cost associated with hires of L workers. As firms do not have the same size in the initial equilibrium, they face different fall in costs, proportional with their size. After this fall in costs, firms are allowed to increase the rent of H workers due to competitive effects. The more L workers are laid off after the increase in the supervision rate, the greater the increase in the rents of H workers is.

However, this result holds if and only if the marginal cost of hiring L workers is positive. This last observation arises from the absence of increase in productivity after the change in supervision rate, in a case where L workers are costless.

This result on skill biased technical change is consistent with the survey of Acemoglu and Autor (2011). In the framework developed here, the relative demand of skills is directly determined by the supervision rate. As such, it provides another approach to the study of skill bias technical change and its interaction with geography. In this spatial model, concurrent with the increase in the rents' skill premium, the decrease of wage gap for H workers is caused by the increase in relative demand for skilled labor.

3.4.3 Increase in t

From a geographic perspective, an increase in t translates to an increase in commute costs for workers.

It is possible to distinguish the impact of an increase in t, either for H or L workers. As it is arguable that time valuation varies between skilled and unskilled workers, this distinction is important and the effect of a change in each parameter will be presented.

Suppose workers face different transportation cost, t_L and t_H. One can use (9) to determine the change in the wage gaps after a change in one of either parameters.

Proposition 6 (Increase in t_H). *A change in t_H has the following effects:*

- *Decrease in rents offered to H workers by firm B , ambiguous effect on firm A:* $\mathrm{d}w_B^H < 0$
- *Increase in wage gap in equilibrium:* $\mathrm{d}\Delta w^H > 0$
- *Increase in the size of firm A:* $\mathrm{d}M(\tilde{x}^H) > 0$

The decrease of the rents given to H by firm B are due to the fall in valuation of the differences in wages by the workers. A greater wage difference is now required to attract the same number of workers. As firm B is less impacted due to its favorable location (at $\tilde{x} = \frac{1}{2}$, she hires more workers than A), it has less incentives to increase the rent it gives to H workers. An important remark is that this result is consistent with empirical observation that an increase in commuting times differences translate to increase in rents differentials, shown in Timothy and Wheaton (2001). Indeed H workers' in firm A higher average commute time is compensated by greater rents.

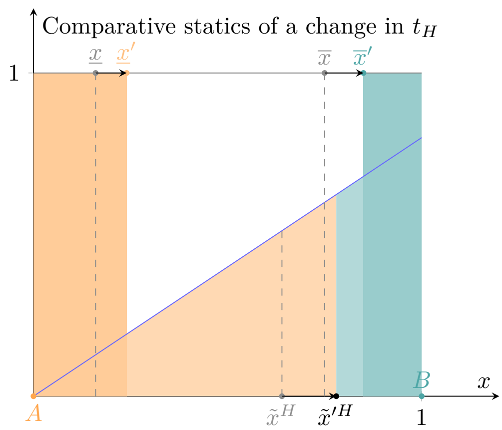

Figure 4: Effects of $\mathrm{d}t_H = 1$, with $M(1) = 0.4,\ \alpha = 2,\ t_L = 1,\ t_H = 2,\ k = 10$
The figure above depicts the change in equilibrium after a positive shock on the commute costs of H workers from 1 to 2.

It is unclear whether the H workers in firm A are worse or better off after this change in commute costs. The first effect of the decrease in wages after the fall in productivity is compensated by the change of sizes of the firms thus leading to ambiguous results.

However, workers in firm B face simultaneously the direct effect from an increase in their commuting costs and the indirect effect of the fall of their rent, decreasing sharply their utility. This paradox arises from the fall in the value of the alternative contract proposed by firm A.

Lastly this result underlines an important idea that completes previous empirical observation by Timothy and Wheaton (2001) . An increase in differences in average commute times is followed by an increase in differences in rents, the fall in competition firms face allow them to lower the rent they offer to H workers.

Proposition 7 (Increase in t_L)**.** *An increase in t_L has the following effects:*

- *A decrease in rents* $\forall j = \{A; B\},\ \mathrm{d}w_j^H < 0$
- *An increase in the wage gap* $\mathrm{d}\Delta w^H > 0$

As the average distance from the employer is greater for L workers in firm B than in firm A, the former, faces a greater increase in its costs. As such it has more incentives to decrease rents of H workers. Consequently the size of B falls and offsets partially the increase in costs it faces. Firm A faces an increase in her costs both from the increase in cost of its initial pool of workers and from the growth in its size. The fall in productivity of H workers arising with the increased cost of the complement translates through fall in rents.

This result allows to determine the role played by the nature of the spatial mismatch on currently employed worker wages.

This global increase in commute costs shifts the spatial mismatch closer to $x = 1$ as the size of firm B decreases. More precisely, some L workers in firm B are now unemployed after the global increase in commute costs. Unable to maintain wages at the previous level firm B prefers to fall in size and lay off workers. H workers laid off then occupy a position in the concurrent firm. The location of the spatial mismatch and its symmetry around $x = 0.5$ is crucial to determine and assess the extent of the wage differences between workers of both same and different skills.

3.4.4 Shift in the distribution of workers M

A change in the distribution of worker will be determined analytically with of a change in the instantaneous rate of increase of the size of the firms, represented by the hazard rate of $M(\cdot)$ for firm A and the reversed hazard rate for firm B .

For simplicity, we will assume that the distribution M switches to W, such that W dominates M in the likelihood ordering, $\frac{W'(x)}{M'(x)}$ is increasing in x. However I also assume that the total mass of H workers is unchanged as to avoid any reduction in total scarcity of this input. This economic implications of this approach are notable and deserve to be explained in the main part of the paper.

This likelihood ordering between $M(\cdot)$ and $W(\cdot)$ describes a displacement of the H workers closer to $x = 1$. The main mechanism driving the results is the increase in the asymmetry of competition the firms face. For instance, firm A has even lower access to H workers under $W(\cdot)$ than $M(\cdot)$, reinforcing her incentive to increase the rent of H workers.

Proposition 8 (Change in $M(\cdot)$)**.** *A change in the distribution to a situation where H workers are located closer to $x = 1$ has the following effects:*

- *The rents offered to H workers by firm A [B] increase [decrease]:* $\mathrm{d}w_A^H > 0,\ \mathrm{d}w_B^H < 0$.
- *Suppose that both distributions are log-concave, the size of firm A (B) is decreasing (increasing):* $\forall\{M(\cdot); W(\cdot)\}$ *both log-concave,* $\mathrm{d}h_A < 0,\ \mathrm{d}h_B > 0$.

- *The rents of L workers in firm A (B) decrease (increase):* $\mathrm{d}w_A^L < 0, \mathrm{d}w_B^L > 0$
- *Thus the wage gaps within firms and between firms increase:* $\mathrm{d}\Delta w^i > 0, \mathrm{d}\Delta w_j > 0$

Where Δw^i is the wage gap between firms for worker $i \in \{L; H\}$ and Δw_j is the wage gap within firm $j \in \{A; B\}$

A shift in the distribution where density of H workers increases close to $x = 1$ leads to greater differences in availability of workers between firms. The firm A is more willing to increase the rents it offers to the workers and firm B is decreasing the rents of H workers.

This result is related to the increase (decrease) of the competitive pressure for firm A (B) after the shift in distribution. A firm facing a greater access to the pool of workers benefits from this situation from two channels.

First, it is less costly to compensate the commute costs of H workers. at constant size after switching from $M(\cdot)$ to $W(\cdot)$ it is less costly to compensate for H workers commuting costs.

Second, This firm faces a faster decrease in the marginal benefit after increasing w_B^H. At constant mass, a greater availability nearby implies that less workers can be met along the rest of the line. As such one can see that firm B has less incentives to increase the rent of H workers. Conversely, both these effects are reversed for firm A, it is now more costly to hire new workers and the firm face a slower decrease in her marginal cost.

More generally, with stochastic ordering properties, it is possible to order and rank various distribution functions. This can be done by using likelihood ordering properties that allow for straightforward variations in both (8a) and (8b), while implying clear results of the shift in the equilibrium. This tool, while being conceptually simple, yields robust and potent outcomes, therefore helping in the formulation and rigorous evaluation of counterfactual scenarios.

4 Concluding Remarks

In this paper, I define a spatial equilibrium of the labor market, in which firms compete for skilled workers. The framework encompasses asymmetries in access to pool of workers and I show simple conditions for existence and uniqueness of equilibrium. More specifically, a firm is assumed to have a better access to the pool of workers available in scarce quantity; this starting point determines the equilibrium characterization and the comparative statics. The complementarity of the labor inputs between skilled and unskilled workers in the production function allows to define relevant rent gaps between and within firms.

The equilibrium characteristics nuance the long-lasting observation that bigger firms tend to offer a positive wage premium. The advantage in access to skilled labor of one firm will allow it to set the rents offered to skilled workers at a lower level, while hiring more workers, in comparison to the competitor. This theoretical model allows to describe the geographic and competitive components driving the rent sharing in this environment. Disentangling the composition and structure of firms allow me to determine factors that allow bigger firms to set lower rents to their workers.

The results on comparative statics also relate to existing results in various strands of the literature but exert different channels. Namely, a skill biased technical progress drives upwards the skill premium and, at the same time, reduces the wage gap between skilled workers of different firms. This result arise from the scarcity of the skilled workers and the competitive pressure firms face. In a case of a global shock, it perfectly translates into an increase of wages of skilled workers by the same amount. For idiosyncratic shocks, the pass through is imperfect and skilled workers do not fully benefit from the increase in productivity. Still the cost of unskilled workers limits the firm in her ability to hire more skilled workers due to the technology of production. Indeed, this paper shows that a firm compensating commute costs of the unskilled workers reduces the rents offered to skilled workers.

Second, a reduction of mobility of skilled workers, through an increase in commute costs, also reduces the rent if they are hired by the bigger firm. This result completes observations on the role of differences in average commute times on wages. The competitive framework increases the dispersion of rents while reducing their level after a fall in mobility.

This model can be further refined by introducing agents' housing decisions, especially important in urban areas where tastes distribution is such that workers given their education enjoy different goods and thus benefit differently from local infrastructures. As a result, introducing a housing market in this model can justify the approach on rent offered rather than nominal wages.

Appendix

This section will provide extensive proofs for the results of the model, in order. For clarity, some expressions will be simplified, in this section $M() = M(\frac{1}{2} + \frac{w_A^H - w_B^H}{2t_H})$.

A Proof of Lemma 1:

The proof that determines the cost of L workers relies on the formulation of commute costs of the firm. As $M(1) < \alpha$, the total mass of L workers employed cannot be greater than $\frac{M(1)}{\alpha}$.

We also know that firm j hires h_j H workers. With the 1 , one can derive the mass of L workers hired: αh_j . The last worker hired by firm j is located at $x = \frac{h_j}{\alpha}$.

Since firms do not compete for L workers, they pay the minimal wage so that the last L worker is indifferent between working or being unemployed. The wage offered by the firm is: $w_j^L = \frac{th_j}{\alpha}$, leading to the following total cost for L workers:

$$w_j^L l_j = \frac{th_j^2}{\alpha^2}$$

This concludes the proof, note that with constant commute costs denoted by c for L workers, the expression becomes linear in mass of H workers hired:

$$w_j^L l_j = \frac{ch_j}{\alpha} \blacksquare$$

B Proof of Lemma 2:

Given 4 we can write the first order conditions of 2a and 2b where all H workers between $[\underline{x}^H; \overline{x}^H]$ are unemployed:

$$k - t\underline{x}^H - \frac{2t}{\alpha^2} M(\underline{x}^H) = \frac{M(\underline{x}^H)}{M'(\underline{x}^H)}$$

$$k - t(1 - \overline{x}^H) - \frac{2t}{\alpha^2}\big(M(1) - M(\overline{x}^H)\big) = \frac{M(1) - M(\overline{x}^H)}{M'(\overline{x}^H)}$$

$$\text{S.t. } \underline{x}^H < \overline{x}^H$$

Note that the existence of such equilibrium relies on **Proposition 2**, proven in D we suppose that the conditions of existence and uniqueness also hold here as I restrict the future results to an unique equilibrium.

The condition for full employment can now be determined when $\underline{x}^H = \overline{x}^H$ implying that the worker located at this precise point is now indifferent between working in either of the firms and being unemployed.

We can now determine a threshold on k , denoted k_{min}, that provides such an equilibrium for any $M(\cdot)$:

$$2k_{min} - t - \frac{2t}{\alpha^2}M(1) = \frac{M(1)}{M'(\underline{x}^H)}$$
$$k_{min} - t(1-\overline{x}^H) - \frac{2t}{\alpha^2}\big(M(1) - M(\overline{x}^H)\big) = \frac{M(1) - M(\overline{x}^H)}{M'(\overline{x}^H)}$$

For any $M(\cdot)$ full-filling these conditions, any increase in k above k_{min} will lead to an increase in wages offered by both firms. As such the worker previously located at $\overline{x}^H$ will now prefer either of the firms rather than not working, consequently, any H worker is now employed, concluding the proof:

$$\forall\ M(\cdot) \text{ log-concave},\ \exists!\ k_{min}, \forall\ 0 < k < k_{min},\ \exists\ \{\underline{x}^H; \overline{x}^H\}\ \in [0;1]^2,\ \underline{x}^H = \overline{x}^H\ \blacksquare$$

C Proof of Proposition 1:

To prove that no corner equilibrium exist we can first look at the case where $w_B^H \geq w_A^H + t$. The following conditions must hold

$$k - w_B^H - \alpha^{-2}tM(1) \geq 0$$
$$k - w_A^H \leq 0$$

The first condition is that of positive profits in equilibrium for firm B. The second identity is a no entry condition for firm A.

Yet we know that $k - w_A^H > k - w_B^H$ since B is the unique employer.
Since $\alpha^{-2}tM(1) \geq 0$. This concludes the proof.

A similar proof can be made when A is the unique employer. ■

D Proof of Proposition 2:

To prove this we need to show that The LHS of 8 are increasing in w_A^H and w_B^H equivalently when

$$\left(\frac{\partial M(\tilde{x}^H)}{\partial w_A^H}\right) - \frac{\partial^2 M(\tilde{x}^H)}{\partial w_A^{H^2}} M(\tilde{x}^H) \geq 0$$
$$\left(-\frac{\partial M(\tilde{x}^H)}{\partial w_B^H}\right)^2 - \left(-\frac{\partial^2 M(\tilde{x}^H)}{\partial w_B^{H^2}}\right)\left(M(1) - M(\tilde{x}^H)\right) \geq 0$$

Given the assumptions of the model, $M(\cdot)$ is convex, as such the second equation always hold. If $M(\cdot)$ is log-concave then the first equation holds

This can be interpreted as a decreasing instantaneous growth of the firm at any location x. Or equivalently, the distance required to increase the current size of the firm by 1% is increasing in x. ■

E Proof of Proposition 3:

First we need to determine the second order conditions. To do so, we only need to check if the profit of firm A is concave.

$$\frac{\partial^2 M(\tilde{x}^H)}{\partial w_A^{H^2}}\Big(k - w_A^H - 2\alpha^{-2}tM(\tilde{x}^H)\Big) - 2\frac{\partial M(\tilde{x}^H)}{\partial w_A^H} - 2\alpha^{-2}t\big(\frac{\partial M(\tilde{x}^H)}{\partial w_A^H}\big)^2 \leq 0 \tag{12a}$$

$$-\frac{\partial^2 M(\tilde{x}^H)}{\partial w_B^{H^2}}\Big(k - w_B^H - 2\alpha^{-2}t\big(M(1) - M(\tilde{x}^H)\big)\Big) - 2\frac{\partial M(\tilde{x}^H)}{\partial w_B^H} - 2\alpha^{-2}t\big(\frac{\partial M(\tilde{x}^H)}{\partial w_B^H}\big)^2 \leq 0 \tag{12b}$$

Given the convexity of $M(\cdot)$, the truly problematic condition is the one of firm A , as $\frac{\partial^2 M(\tilde{x}^H)}{\partial w_B^{H^2}}$ is positive, that automatically full-fills the second order condition for firm B. One can study the second order condition locally at the equilibrium. Given (8a) and (8b) one can study locally (12).

Replacing $k - w_A^H - 2\alpha^{-2}tM(\tilde{x}^H)$ $\Big($and $k - w_B^H - 2\alpha^{-2}t\big(M(1) - M(\tilde{x}^H)\big)$ $\Big)$ by $\frac{M(\tilde{x}^H)}{\frac{\partial M(\tilde{x}^H}{\partial w_A^H}}$, $\Big($ resp. $\frac{\big(M(1)-M(\tilde{x}^H)\big)}{-\frac{\partial M(\tilde{x}}{\partial w_B^H}}\Big)$ allows us to prove that log-concavity is a sufficient but not necessary condition for the candidate to be a maximum.

As Such we can already prove that the interior solution is a maximum.

The next step of the proof, by contradiction allows us to show that in equilibrium, $w_B^H < w_A^H$

Assuming that in equilibrium $w_B^H \geq w_A^H$ then:

$$k - w_B^H - 2\alpha^{-2}t\big(M(1) - M(\tilde{x}^H)\big) < k - w_A^H - 2\alpha^{-2}tM(\tilde{x}^H)$$
$$M(1) - M(\tilde{x}^H) > M(\tilde{x}^H)$$

Given (8a) and (8b), we come to a contradiction that both equations cannot hold when $M(1) - M(\tilde{x}^H) > M(\tilde{x}^H)$. This concludes the proof and allow us to say that in equilibrium $w_A^H > w_B^H$.

Finally we will prove the statement that firm B hires more H workers than A in equilibrium by contradiction

Assuming that $M(\tilde{x}^H) > M(1) - M(\tilde{x}^H)$ and since $w_A^H > w_B^H$, we have the following:

$$k - w_A^H - \frac{2t}{\alpha^2} M(\tilde{x}^H) < k - w_B^H - \frac{2t}{\alpha^2}\big(M(1) - M(\tilde{x}^H)\big)$$

However this inequality violates the first order conditions, as $\frac{M(\tilde{x}^H)}{M'(\tilde{x}^H)} > \frac{M(1)-M(\tilde{x}^H)}{M'(\tilde{x}^H)}$. This leads to a contradiction, implying that $M(\tilde{x}^H) < M(1) - M(\tilde{x}^H)$ and $w_A^L < w_B^L$ by equivalence in equilibrium. ■

F Proofs of Comparative Statics

F.1 Proof of Proposition 4:

The total derivative of (11) with respect to Δk is:

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}\Delta k} = \frac{\mathrm{d}\Delta w^H}{\mathrm{d}\Delta k}\left(\frac{\partial \frac{M(1)-M()}{M'()}}{\partial \Delta w^H} - \frac{\partial \frac{M()}{M'()}}{\partial \Delta w^H} - \frac{4t}{\alpha^2}\frac{\partial M()}{\partial \Delta w^H}\right) + 1$$

Thus one has,

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}\Delta k}\left(\frac{\partial \frac{M(1)-M()}{M'()}}{-\partial \Delta w^H} + \frac{\mathrm{d}\frac{M()}{M'()}}{\mathrm{d}\Delta w^H} + \frac{4t}{\alpha^2}\frac{\mathrm{d}M()}{\mathrm{d}\Delta w^H}\right) = 1$$

Since $\frac{\partial \frac{M(1)-M()}{M'()}}{-\partial \Delta w^H}$ is positive due to the convexity of $M(\cdot)$ and $\frac{\partial \frac{M()}{M'()}}{\partial \Delta w^H}$ is positive due to the log-concavity of $M(\cdot)$. The effect of an increase in the productivity gap between firms translates in an increase in the rent for H workers. ■

F.2 Proof of Proposition 5:

The effect of an increase in α on the wage gap of H workers is the following

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}\alpha} = \frac{\mathrm{d}\frac{M(1)-2M()}{M'()}}{\mathrm{d}\alpha} + \frac{\mathrm{d}\frac{2t_L\left(M(1)-2M()\right)}{\alpha^2}}{\mathrm{d}\alpha}$$

Using the total derivative and the chain rule yields

$$\frac{\mathrm{d}\frac{M(1)-2M()}{M'()}}{\mathrm{d}\alpha} = \frac{\partial\frac{M(1)-2M()}{M'()}}{\partial\Delta w^H}\frac{\mathrm{d}\Delta w^H}{\mathrm{d}\alpha}$$
$$\frac{\mathrm{d}\frac{2t_L\left(M(1)-2M()\right)}{\alpha^2}}{\mathrm{d}\alpha} = \frac{\partial\frac{2t_L\left(M(1)-2M()\right)}{\alpha^2}}{\partial\Delta w^H}\frac{\mathrm{d}\Delta w^H}{\mathrm{d}\alpha} + 2t_L\big(M(1)-2M()\big)\frac{-2}{\alpha^3}$$

With that we can determine the effect of an increase in α on the wage gap

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}\alpha}\left(1+\frac{4t_L}{\alpha^2}M'() + \frac{2M'()^2 + M''()\big(M(1)-2M()\big)}{M'()^2}\right) =$$
$$2t_L\big(M(1)-2M()\big)\frac{-2}{\alpha^3} \ \blacksquare$$

F.3 Proof of Proposition 6:

The effect of an increase in t_H on the wage gap is the following

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_H} = \frac{\mathrm{d}\frac{M(1)-2M(\frac{1}{2}+\frac{\Delta w^H}{2t_H})}{\frac{1}{2t_H}M'(\frac{1}{2}+\frac{\Delta w^H}{2t_H})}}{\mathrm{d}t_H} + \frac{-4t_l}{\alpha^2}\left(\frac{\mathrm{d}M(\frac{1}{2}+\frac{\Delta w^H}{2t_H})}{\mathrm{d}t_H}\right)$$

Using the chain rule

$$\frac{\mathrm{d}\frac{M(1)-2M()}{\frac{1}{2t_H}M'()}}{\mathrm{d}t_H} = 2t_H\left(\frac{-2M'()^2 - M''()\big(M(1)-2M()\big)}{M'()}\right)\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_H}$$
$$-2t_H\frac{-\frac{\Delta w^H}{2{t_H}^2}\Big(-2M'()^2 - M''()\big(M(1)-2M()\big)\Big)}{M'()}$$
$$+2\frac{M(1)-2M()}{M'()}$$
$$\frac{\mathrm{d}M()}{\mathrm{d}t_H} = M'()\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_H} - \frac{\Delta w^H}{2{t_H}^2}M'()$$

Now plugging both expressions in $\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_H}$ yields

$$\begin{aligned}
&\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_H}\Big(1+\frac{4t_L}{\alpha^2}M'()+\frac{2M'()^2+M''()\big(M(1)-2M()\big)}{M'()^2}\Big)=\\
&-\frac{\Delta w^H}{2{t_H}^2}\Big(-\frac{4t_l}{\alpha^2}M'()-2t_H\frac{2M'()^2+M''()\big(M(1)-2M()\big)}{M'()^2}\Big)\\
&-\frac{-1}{2t_H}\frac{M(1)-2M()}{M'()}
\end{aligned}$$

Finally, due to log-concavity of $M(\cdot)$ this concludes the proof and the impact on the wage gap between H workers is positive. As the wage gap between H workers increases, the difference in sizes between firms falls.

One can determine the effect on each wage offered by the firm to H workers

$$\begin{aligned}
-\frac{\mathrm{d}w_A^H}{\mathrm{d}t_H}&=\frac{\mathrm{d}\ 2t_H\frac{M()}{M'()}}{\mathrm{d}t_H}+\frac{2t_L}{\alpha^2}\frac{\mathrm{d}M()}{\mathrm{d}t_H}\\
-\frac{\mathrm{d}w_B^H}{\mathrm{d}t_H}&=\frac{\mathrm{d}\ 2t_H\frac{\left(M(1)-M()\right)}{M'()}}{\mathrm{d}t_H}+\frac{2t_L}{\alpha^2}\frac{\mathrm{d}\big(M(1)-M()\big)}{\mathrm{d}t_H}
\end{aligned}$$

Which can be rewritten as follows

$$\begin{aligned}
-\frac{\mathrm{d}w_A^H}{\mathrm{d}t_H}&=2\frac{M()}{M'()}+\frac{-\Delta w^H}{2t_H^2}\Big(2t_H\frac{M'()^2-M''()M()}{M'()^2}+\frac{2t_L}{\alpha^2}M'()\Big)+\\
&\quad\frac{\mathrm{d}\ \Delta w^H}{\mathrm{d}t_H}\Big(\frac{\partial\frac{M()}{M'()}}{\partial\Delta w^H}+\frac{\partial M()}{\partial\Delta w^H}\Big)\\
-\frac{\mathrm{d}w_B^H}{\mathrm{d}t_H}&=2\frac{M(1)-M()}{M'()}+\frac{-\Delta w^H}{2t_H^2}\Big(2t_H\frac{-M'()^2-M''()\big(M(1)-M()\big)}{M'()^2}-\frac{2t_L}{\alpha^2}M'()\Big)+\\
&\quad\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_H}\Big(2t_H\frac{\partial\frac{M(1)-M()}{M'()}}{\partial\Delta w^H}-\frac{2t_L}{\alpha^2}\frac{\partial M()}{\partial\Delta w^H}\Big)
\end{aligned}$$

So the effect on B is unambiguous and the rents she offers to H workers decrease. The effect for firm A individual rents she offers to H workers is ambiguous. Two partial equilibrium effects can be described. First the increase in commuting cost for H workers decreases the marginal gain of w_J^H, $\forall j$. Associated with this first effect, the second one impacts the role of differences in wages, as the indifferent is less sensitive to differences in wages, it naturally shifts towards $\frac{1}{2}$. This creates incentives for firm A to increase wages, this is reversed for firm B.

The sum of those effects is greater for firm B, so w_B^H decreases more than w_A^H, shifting composition of firms and offsetting the initial effects. ■

F.4 Proof of Proposition 7:

In the same spirit of the previous proof, the effect of t_L on the H workers wage gap is

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_L} = \frac{\mathrm{d}\frac{M(1)-2M()}{M'()}}{\mathrm{d}t_L} + \frac{\mathrm{d}\frac{2t_L\big(M(1)-2M()\big)}{\alpha^2}}{\mathrm{d}t_L}$$

The impact of a change in t_L on the wage gap between H workers is the following

$$\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_L}\left(1 + 4\frac{t_L}{\alpha^2}M'() + \frac{2M'()^2 + M''()\big(M(1) - 2M'()\big)}{M'()^2}\right) = \frac{2}{\alpha^2}\big(M(1) - 2M()\big)$$

The effect of a change in t_L has the following effects on the individual rents

$$\begin{aligned}
\frac{\mathrm{d}w_A^H}{\mathrm{d}t_L} &= \frac{-2M()}{\alpha^2} - \Big(\frac{2t_L}{\alpha^2}\frac{\partial M()}{\partial w_A^H} + \frac{\partial \frac{M()}{M'()}}{\partial w_A^H}\Big)\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_L} \\
\frac{\mathrm{d}w_B^H}{\mathrm{d}t_L} &= \frac{-2\big(M(1) - M()\big)}{\alpha^2} + \frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_L}\left(\frac{2t_L}{\alpha^2}\frac{M()}{\partial w_B^H} - \frac{\partial\frac{M(1)-M()}{M'()}}{\partial w_A^H}\right)
\end{aligned}$$

The first effect of the RHS is the mechanic effect on the rents of an increase in the transport cost, it is always more detrimental to firm B due to the original differences in size between firms. The second effect, is a mitigation effect after a change in wage gap, it is the supplementary cost after an increase in employment of L workers. It can be interpreted as a transfer between firms. Finally, there is another mitigation effect due to the change of employment of H workers, however the sum of this effect for both firms doesn't cancel out

.

Indeed summing both changes yields

$$\frac{\mathrm{d}w_A^H}{\mathrm{d}t_L} + \frac{\mathrm{d}w_B^H}{\mathrm{d}t_L} = \frac{-2M(1)}{\alpha^2} + M(1)\frac{M''()}{M'()^2}\frac{\mathrm{d}\Delta w^H}{\mathrm{d}t_L}$$

As we can see there is a downward push by the increase in labor cost of L workers. However it is partially offset by the sum of the inverse of hazard and reversed hazard rates. The greater the sum, the lower the decrease in rents offered to L workers. ■

F.5 Proof of Propositon 8:

An increase in the likelihood order leads an increase in both the hazard rate and reverse hazard rate orders Let $W(\cdot)$ be the new distribution that has higher hazard rate and reverse hazard rate orders. By definition of hazard rate [reversed hazard rate] ordering

$$\frac{W'(x)}{W(1) - W(x)} \leq \frac{M'(x)}{M(1) - M(x)}$$
$$\frac{W'(x)}{W(x)} \geq \frac{M'(x)}{M(x)}$$

In addition to that, Shaked and Shanthikumar (2007) provide elements to complete the proof, namely the likelihood ordering implying the usual ordering: Finally, W is higher than M in the usual order, $\forall x,\ W(x) \geq_{ST} M(x)$ and $\exists A \neq \emptyset,\ \forall x \in A,\ W(x) < M(x)$

Using all inequalities implied by stochastic ordering allows to prove the results in individual rents. Firm B lowers the wage she offers to H workers and firm A increases the rents she offers to H workers, .

Lastly we can determine the dynamics of the change in firm sizes. By contradiction we can assess that firm A has a non increasing size after the shift in the distribution.

First Suppose that A has the same size under W than M, then $\exists\ \hat{x_H},\ W(\hat{x_H}) = M(\hat{x_H})$ and since $W >_{ST} M$, then $1 > \hat{x_H} > \tilde{x} > 0$. Using both sets of first order conditions, we have the following identity.

$$w_A - w_A' = \frac{W(\hat{x_H})}{W'(\hat{x_H})} - \frac{M(\tilde{x}_H)}{M'(\tilde{x}_H)}$$
$$w_B - w_B' = \frac{W(1) - W(\hat{x_H})}{W'(\hat{x_H})} - \frac{M(1) - M(\tilde{x}_H)}{M'(\tilde{x}_H)}$$

Using the individual changes in wage setting in equilibrium we have the following. $w_A' > w_A$ and $w_B' < w_B$. As it is assumed that $M(\tilde{x_H}) = W(\hat{x_H})$, we come to a contradiction.

To extend the result for $M(\tilde{x}_H) < W(\hat{x}_H)$, one just need to make the (reasonable) assumption that $W(\cdot)$ is log-concave.

Given that, starting from $M(\tilde{x_H}) = W(\hat{x_H})$, and increasing w_A' (decreasing w_B') by $\epsilon > 0$, it leads to an increase in $\frac{W(\hat{x_H})}{W'(\hat{x_H})}$ (decrease in $\frac{W(1)-W(\hat{x}_H)}{W'(\hat{x}_H)}$) as $W(\cdot)$ is log-concave, leading to a contradiction. ■

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